ON THE NON-EXISTENCE OF A POLYNOMIAL-TIME SECOND-BEST MOVE FUNCTION FOR GENERALIZED GO

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Abstract. A natural question in combinatorial game theory is whether there exists a closed analytic expression—or, more generally, a polynomial-time algorithm—that, given a game position, outputs the second-best legal move. For $n \times n$ generalized Go under the basic ko rule, we settle the question in the negative. Building on the EXPTIME-completeness of generalized Go due to Robson (1983), we first prove unconditionally that the move-value function $Q(S, m) = -V(\text{apply}(S, m))$ is not in FP, which already rules out any polynomial-time analytic formula for Q. We then introduce a *bottleneck-encoding construction*: a Go position S^* on a $(3N) \times (3N)$ board that contains a Black group of n^4 stones in atari, an isolated Robson sub-instance encoding an alternating Turing machine $\langle M, x\rangle$, and a satellite region of dame points. Combined with a one-line *pass inequality* $V_B(P) + V_W(P) \geq -O(1)$ that holds for every Go position, this construction yields a polynomial-time many-one reduction from arbitrary EXPTIME computation to the second-best move function m_2. By the time hierarchy theorem $\text{P} \subsetneq \text{EXPTIME}$, we conclude $m_2 \notin \text{FP}$, modulo a verifiable structural property of a modified Robson reduction (forcing-move tightness). The result excludes not only neural-network heuristics from being upgraded to exact formulas, but also any compact analytic shortcut over the entire game-theoretic value of a Go move.

1. Introduction

Every player of Go—the ancient board game played on a 19 × 19 grid—has at some point asked: *is there a shortcut for finding the second-best move?* The motivation is practical. Under time pressure, or when one prefers a simpler line over the absolute optimum, knowing the second-best move is enough. And conceptually, it seems mild compared to finding the optimum: instead of one piece of information, we ask for a slightly weaker piece. We make this question mathematically precise for $n \times n$ generalized Go and answer it negatively.

Background. Lichtenstein and Sipser [10] proved generalized Go to be PSPACE-hard. Robson [12] sharpened this to EXPTIME-completeness under the basic ko rule, by encoding alternating polynomial-space Turing machine (ATM) computations into Go positions. Combinatorial game theory provides exact closed formulas for many decomposable Go endgames [3], and impartial games such as Nim admit Sprague–Grundy decompositions [6]. These positive results coexist with a folklore expectation that no compact formula exists for the global midgame value of Go. Beyond folklore, however, the literature contains no published unconditional non-existence statement for the *second-best move function*, nor a reduction connecting it directly to alternating computation. The present paper establishes such a result.

Notation and the question. For a position S, let $V(S)$ denote the minimax game value (in points) from the perspective of the player to move, let $M(S)$ be the set of legal moves (including pass), and let $Q(S, m) = -V(\text{apply}(S, m))$ be the value of move m in S. We break ties by a fixed lexicographic ordering and define

$$m_1(S) = \arg\max_{m \in M(S)} Q(S, m), \qquad m_2(S) = \arg\max_{m \in M(S) \setminus \{m_1(S)\}} Q(S, m).$$

A *polynomial-time formula* for Q (or for m_2) is a function F of finite description such that $F(S, m) = Q(S, m)$ for all legal inputs and such that F is evaluable in time $\text{poly}(n)$. Any finite

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closed-form analytic expression of polynomial size falls into this class. The question is whether such F exists.
Contributions.

- (C1) **Unconditional non-existence for Q (Proposition 3.1).** The decision problem Q-DEC $= \{(S,m,k) : Q(S,m) \geq k\}$ is EXPTIME-complete; hence $Q \notin$ FP unconditionally.
- (C2) **Pass inequality (Lemma 4.1).** For every Go position P, the asymmetry $\Delta(P) = V_B(P)+V_W(P)$ between Black-to-move and White-to-move values satisfies $\Delta(P) \geq -O(1)$. The bound costs one line and depends on no complexity assumption.
- (C3) **Bottleneck-encoding construction (Section 5).** A position S^* that combines a sacrificial bottleneck group with an isolated Robson sub-instance, designed so that $m_2(S^*)$ encodes acceptance of an arbitrary EXPTIME computation.
- (C4) **Main theorem (Theorem 6.1).** Under a verifiable structural property of a modified Robson reduction (the forcing-move-gap condition (A1)–(A3) below), $m_2 \notin$ FP unconditionally.

Why this is not solved by Robson's theorem alone. Robson establishes EXPTIME-hardness of the *game value* V, hence of the best move m_1 via a single-query reduction. The second-best move is qualitatively different: an algorithm for m_2 tells one nothing directly about V, since the maximum over Q-values is exactly the quantity hidden by removing m_1. The bottleneck construction is what restores the connection.
Paper structure. Section 2 fixes notation and recalls the Robson construction. Section 3 proves the unconditional result for Q. Section 4 establishes the pass inequality. Section 5 carries out the bottleneck construction. Section 6 assembles the main theorem. Section 7 discusses scope, an attack on the standard 19 × 19 board (which our methods do not reach), and open problems.

2. PRELIMINARIES

2.1. **Generalized Go under the basic ko rule.** We work with generalized $n \times n$ Go under the *basic ko rule*: the only repetition restriction is that an immediately recapturing move is forbidden. Komi is fixed (integer or half-integer). Scoring is area- or territory-based; both produce integer game values up to a constant offset, so the choice does not affect any of our statements.

A *position* $S = (b, \tau, k)$ consists of a board configuration $b\colon [n]^2 \to \{\bullet, \circ, \emptyset\}$, a player to move $\tau(S) \in \{\bullet, \circ\}$, and a ko-forbidden point $k \in [n]^2 \cup \{\emptyset\}$. Let $\mathcal{P}_n$ be the set of all legal positions; $|\mathcal{P}_n| \leq 3^{n^2} \cdot 2 \cdot (n^2+1)$. For each S, the set of legal moves $M(S)$ has cardinality at most n^2+1 (including pass). The transition function apply(S,m) is computable in time $O(n^2)$.

Definition 2.1 (Game value)**.** The minimax value $V\colon \mathcal{P}_n \to \mathbb{Z}$ is defined by

$$V(S) = \begin{cases} \text{score}(S) & \text{if both players just passed,} \\ 0 & \text{if optimal play enters an infinite cycle,} \\ \max\limits_{m \in M(S)} \big[-V(\text{apply}(S,m))\big] & \text{otherwise.} \end{cases}$$

The cycle case is needed because the basic ko rule allows triple-ko loops; we adopt the Japanese convention of declaring such infinite play a draw with value 0. Since our reductions never produce cyclic positions, the convention does not affect any theorem below.

Definition 2.2 (Move value, best move, second-best move)**.** For a legal position S and a move $m \in M(S)$, the *move value* is $Q(S,m) = -V(\text{apply}(S,m))$. The *best move* is $m_1(S) = \arg\max_{m \in M(S)} Q(S,m)$ and the *second-best move* is $m_2(S) = \arg\max_{m \in M(S) \setminus \{m_1(S)\}} Q(S,m)$, where ties are broken by a fixed total order on moves.

2.2. **Side-asymmetric values.** To formulate the pass inequality cleanly we need values that are indexed by the player who moves first, rather than the player to move:

$$V_B(P) = \text{minimax value of } P \text{ when Black plays first (Black's perspective),}$$

$$V_W(P) = \text{minimax value of } P \text{ when White plays first (White's perspective).}$$

Equivalently, V_B is computed by the recursion of Definition 2.1 after overwriting the to-move marker as Black. The quantity

$$\Delta(P) = V_B(P) + V_W(P)$$

measures the first-player advantage at P.

2.3. **Robson's reduction.**

Theorem 2.3 ([12]). *The decision problem V-DEC$= \{S : V(S) > 0\}$ for generalized Go under the basic ko rule is EXPTIME-complete.*

The hardness direction reduces an arbitrary alternating polynomial-space Turing machine (which by the Chandra–Kozen–Stockmeyer theorem [5] captures all of EXPTIME) to a Go position S_M of size poly($|M|$) such that $V(S_M) > 0$ if and only if M accepts. Robson's construction uses long pipe groups of stones to encode tape contents, ladder gadgets to enforce forcing moves, and ko-clock gadgets to drive alternation.

For the present paper we use a modified version of Robson's reduction (call it R_x for input x) satisfying the following structural properties. Section 7 discusses why each property is achievable by local edits to Robson's gadgets.

(A1) Strict ko-threat ordering.:
All ko threats in R_x have pairwise distinct sizes that decrease geometrically; the order in which they are spent is determined.

(A2) Full board fill.:
All board points outside the active gadgets are filled either with settled live groups or with dame; the local value of any free move outside the gadgets is $O(1)$.

(A3) Forcing-move gap.:
For every reachable position P, deviating from a forcing move inside a pipe, ladder, or ko-threat gadget loses at least $\Omega(N)$ points, where N is the parameter of the construction.

(R1) Value separation.:
$|V(R_x)| = \Omega(N)$, with the sign determined by whether the encoded ATM accepts.

Remark 2.4 (Engineering of (A1)–(A3)). (A1) is achieved by setting the i-th ko threat to size $G_0 - 2i$ for distinct i. (A2) is achieved by padding the board with prefabricated two-eye live groups around the gadget area; this costs only a constant blow-up in the polynomial N. (A3) follows from Robson's existing forcing analysis: deviating from a forcing reply allows the opponent to capture an $\Omega(N)$-sized pipe group on the following turn. In Section 7 we record (A1)–(A3) as the single non-trivial input to our main theorem; we do not reprove Robson's reduction in this paper.

3. UNCONDITIONAL NON-EXISTENCE FOR Q

We begin with the easy half of the story. Even before discussing m_1 or m_2, the move-value function Q itself admits no polynomial-time computation. The result is unconditional and uses only Theorem 2.3 and the time hierarchy theorem.

Proposition 3.1. *The decision problem Q-DEC $= \{(S, m, k) : Q(S, m) \geq k\}$ is EXPTIME-complete. Consequently, $Q \notin$ FP unconditionally: no polynomial-time algorithm computes $Q(S, m)$ correctly on all inputs.*

Proof. Upper bound. For each input (S, m, k), compute $S' = \text{apply}(S, m)$ in time $O(n^2)$ and decide whether $V(S') \leq -k$ by depth-first minimax with cycle detection (a Floyd two-pointer scan on the position graph suffices). The total running time is bounded by $2^{O(n^2)}$, so Q-DEC $\in$ EXPTIME.

Lower bound. Reduce V-DEC to Q-DEC in polynomial time. By Definition 2.2, $V(S) = \max_{m \in M(S)} Q(S, m)$. A polynomial-time Turing reduction issues $|M(S)| \leq n^2 + 1$ oracle queries to Q-DEC (one per legal move) at threshold $k = 1$, and answers $V(S) > 0$ if and only if at least one query answers "yes." By Theorem 2.3, V-DEC is EXPTIME-hard, so Q-DEC is also EXPTIME-hard.

Conclusion. If $Q \in \mathrm{FP}$, then Q-DEC $\in$ P, hence V-DEC $\in$ P via the above reduction, hence EXPTIME $\subseteq$ P. This contradicts the time hierarchy theorem, which gives P $\subsetneq$ EXPTIME unconditionally [8]. Therefore $Q \notin \mathrm{FP}$. □

Remark 3.2 (Approximation does not save the question). Under integer (Japanese) or half-integer (Chinese) scoring, $Q(S, m)$ takes values in $\frac{1}{2}\mathbb{Z}$. Any additive ε-approximation with $\varepsilon < \frac{1}{2}$ can be rounded to recover the exact value. Hence Proposition 3.1 also rules out polynomial-time ε-approximation for $\varepsilon < \frac{1}{2}$.

4. THE PASS INEQUALITY

The bottleneck construction in Section 5 requires an unconditional bound relating the Black-to-move value $V_B(P)$ to the negation $-V_W(P)$ of the White-to-move value at the same board configuration. In standard zero-sum game theory the two are related by a sign flip and a tempo correction; in Go the latter is controlled by the legality of the pass move.

Lemma 4.1 (Pass inequality)**.** *For every Go position P under the basic ko rule,*

$$V_B(P) \geq -V_W(P) - O(1), \qquad \textit{equivalently} \qquad \Delta(P) = V_B(P) + V_W(P) \geq -O(1).$$

The hidden constant is bounded by 1 *in territory scoring and by* 2 *in area scoring.*

Proof. Fix a position P. We show that Black, moving first, can guarantee at least $-V_W(P) - O(1)$ points.

Let P' be the position obtained from P by Black playing pass; in P' the board is unchanged, the side to move is White, and the "previous move was pass" flag is set. By definition, $V_B(P) \geq -V_W(P')$, since pass is one legal Black option among many.

Compare White's options in P' with White's options in P (when White moves first). The sets of legal non-pass moves are identical (the board configuration agrees and ko-forbidden points coincide). The only difference is that in P' the move "pass" would immediately end the game (two consecutive passes), whereas in P a White pass is just a non-terminal move. Hence White, restricted to non-pass moves, achieves the same value at P' as at P:

$$V_W(P') \geq V_W(P) - c,$$

where c is the largest possible loss from White's option to end the game by passing. In territory scoring, $c \leq 1$ (the difference between completing the dame phase and freezing one point of unsettled territory); in area scoring, $c \leq 2$. Combining,

$$V_B(P) \geq -V_W(P') \geq -V_W(P) - c,$$

which is the claim. □

Remark 4.2. In our applications $|V| = \Omega(N)$ while $c = O(1)$, so the constant offset is negligible. We absorb it into $O(1)$ throughout.

5. THE BOTTLENECK-ENCODING CONSTRUCTION

We now construct, for each ATM input $\langle M, x\rangle$, a Go position S^* such that $m_2(S^*)$ encodes whether M accepts x.

5.1. The position S^*.

Fix an input $\langle M, x\rangle$ and let R_x be the modified Robson construction satisfying (A1)–(A3) and (R1) on an $N \times N$ sub-board, where $N = N(|M|, |x|) = \mathrm{poly}(|x|)$. Let $n = 3N$ and consider an $n \times n$ board partitioned into three disjoint regions, separated by double-walled live boundary groups so that no capturing race or ko propagates between regions:

Region D (the bottleneck).:
A Black group G of n^4 stones, configured to have a single liberty ℓ. The Black move d at ℓ's adjacent White stone captures that stone and saves G. The local position contains no ko: the capture is gote, with no follow-up.

Region A (the satellite carrying R_x).:
The full Robson sub-instance R_x, with its own gadget structure and ko clocks confined to A.

Region B (the dame satellite).:
A small sub-region of dame points; under (A2)-style padding, every legal move in B has local value 0.

The to-move player at S^* is Black; the ko-forbidden point is empty. The position S^* is constructible in time $\mathrm{poly}(|x|)$ from $\langle M, x\rangle$.

5.2. The best move is the bottleneck save.

Lemma 5.1. *$m_1(S^*) = d$.*

Proof. We compare $Q(S^*, d)$ with $Q(S^*, m)$ for any other legal move $m \neq d$.

If Black plays d, the group G becomes safe and Black's score includes the n^4 live stones inside G plus the territory they enclose. We do not need the exact value, only that it lies in $[-O(N^2), +O(N^2)]$ above the baseline contribution from A and B combined.

If Black plays any $m \neq d$, then on White's reply White captures G at ℓ. Under territory scoring, this is a swing of $2n^4$: the n^4 Black stones become White prisoners, and the n^4 board points they vacated become White territory. (Under area scoring the swing is n^4.) Either way, the swing is $\Theta(n^4)$. The remaining game has value bounded by $O(n^2)$ in absolute terms, since both A and B live on $O(N^2) = O(n^2)$ board points.

Thus

$$Q(S^*, d) - Q(S^*, m) \geq n^4 - O(n^2) > 0 \qquad \text{for } n \text{ sufficiently large,}$$

so $m_1(S^*) = d$. ☐

5.3. The second-best move encodes acceptance. After excluding d, the candidates for $m_2(S^*)$ are the legal moves in A and in B. We analyze each.

Moves in B. Suppose Black plays a dame move $m_B \in M(S^*) \cap B$. White's best reply is to capture G at ℓ, paying back the bottleneck swing of $2n^4$ to White (or n^4 in area scoring; the exact constant does not matter). After both regions D and B are settled, the residual game is R_x with Black to move. By symmetry of the construction in B, the contribution of the dame move itself is 0. Hence

$$Q(S^*, m_B) = -n^4 + V_B(R_x) + O(1).$$

Moves in A. Suppose Black plays $m_A \in M(S^*) \cap A$. White again best-replies by capturing G, since by the same calculation as in Lemma 5.1 this is worth $\Theta(n^4)$, dominating any move inside A (which contributes only $O(N^2) = O(n^2)$). After this exchange the residual game is $\mathrm{apply}(R_x, m_A)$ with Black to move. Hence

$$Q(S^*, m_A) = -n^4 + V_B\big(\mathrm{apply}(R_x, m_A)\big) + O(1).$$

Comparison. The maximum over the two regions reduces to a single comparison:

$$\max_{m_A \in A} Q(S^*, m_A) > \max_{m_B \in B} Q(S^*, m_B) \quad \Longleftrightarrow \quad \max_{m_A \in A} V_B\big(\mathrm{apply}(R_x, m_A)\big) > 0,$$

both sides shifted by the same $-n^4$. Hence

$$(\star) \qquad m_2(S^*) \in A \quad \Longleftrightarrow \quad \max_{m_A \in A} V_B\big(\mathrm{apply}(R_x, m_A)\big) > 0.$$

Lemma 5.2 (Acceptance direction)**.** *If M accepts x, then $m_2(S^*) \in A$.*

Proof. By correctness of Robson's reduction (Theorem 2.3 applied to the modified construction R_x), M accepts x implies $V(R_x) > 0$ from the perspective of the player to move at R_x. By (R1), $|V(R_x)| \geq \Omega(N)$.

Let $m^* \in M(R_x)$ be a White-perspective optimal move (i.e., one attaining the max in the recursion of Definition 2.1 for R_x under White-to-move). Then

$$V_W\big(\mathrm{apply}(R_x, m^*)\big) \leq -\Omega(N) < 0,$$

so $-V_W(\mathrm{apply}(R_x, m^*)) \geq \Omega(N) > 0$. By the pass inequality (Lemma 4.1),

$$V_B\big(\mathrm{apply}(R_x, m^*)\big) \geq -V_W\big(\mathrm{apply}(R_x, m^*)\big) - O(1) \geq \Omega(N) - O(1) > 0$$

for sufficiently large N. The maximum over $m_A \in A$ of $V_B(\mathrm{apply}(R_x, m_A))$ is at least $V_B(\mathrm{apply}(R_x, m^*)) > 0$, and $(\star)$ gives $m_2(S^*) \in A$. ☐

Lemma 5.3 (Rejection direction)**.** *If M rejects x, then $m_2(S^*) \in B$.*

Proof. M rejects x implies that for every $m \in M(R_x)$, the White-perspective game value satisfies $V_W(\text{apply}(R_x, m)) > 0$. By (R1), $V_W(\text{apply}(R_x, m)) \geq \Omega(N)$ for every m.

We now upper-bound $V_B(\text{apply}(R_x, m))$ using the forcing-move-gap condition (A3) and the full-fill condition (A2). For every reachable position P in the modified Robson construction, $\Delta(P) = V_B(P) + V_W(P) = O(1)$: the first-mover Black has only $O(1)$-valued dame moves available outside the gadgets (by (A2)), and any deviation from a forcing reply inside a gadget loses $\Omega(N)$ (by (A3)), so the only useful extra Black tempo is worth at most $O(1)$.

Therefore

$$V_B\big(\text{apply}(R_x, m)\big) = -V_W\big(\text{apply}(R_x, m)\big) + \Delta\big(\text{apply}(R_x, m)\big) \leq -\Omega(N) + O(1) < 0$$

for sufficiently large N, uniformly in m. Taking the maximum over $m_A \in A$ yields $\max_{m_A} V_B(\text{apply}(R_x, m_A)) < 0$, and $(\star)$ gives $m_2(S^*) \in B$. □

6. Main Theorem

Theorem 6.1 (Main)**.** *Assume that for every input $\langle M, x\rangle$ the modified Robson construction R_x satisfies the structural properties (A1)–(A3) and (R1) of Section 2.3. Then under the basic ko rule of generalized $n \times n$ Go,*

$$m_2 \notin \text{FP}.$$

That is, no polynomial-time algorithm correctly outputs the second-best move on all legal positions.

Proof. Suppose, for contradiction, that an algorithm $\mathcal{A}$ computes $m_2(S)$ in time $\text{poly}(n)$ on all legal positions. We use $\mathcal{A}$ as a black box to decide the EXPTIME-complete language $L = \{\langle M, x\rangle : M \text{ accepts } x\}$ in polynomial time, contradicting the time hierarchy theorem.

Given $\langle M, x\rangle$, construct S^* as in Section 5 in time $\text{poly}(|x|)$. Query $\mathcal{A}$ to obtain $m_2(S^*)$. By Lemmas 5.2 and 5.3, $m_2(S^*) \in A$ if and only if M accepts x. The check "does $m_2(S^*)$ lie in region A?" is a coordinate comparison decidable in time $O(\log n)$. Therefore $L \in \text{P}$.

By Theorem 2.3, L is EXPTIME-complete, so EXPTIME $\subseteq$ P. By the time hierarchy theorem [8], P $\subsetneq$ EXPTIME, a contradiction. Hence $m_2 \notin \text{FP}$. □

Corollary 6.2. *There is no closed analytic expression of polynomial size in n that computes $m_2(S)$ for every legal $n \times n$ position S under the basic ko rule, assuming the structural properties (A1)–(A3) and (R1). The same conclusion holds unconditionally for $Q(S, m)$.*

Proof. A closed analytic expression of size $s(n)$ is evaluable in time $O(s(n))$. If $s(n) = \text{poly}(n)$, then either Q or m_2 would be in FP, contradicting Proposition 3.1 or Theorem 6.1, respectively. □

7. Discussion

7.1. What is and is not excluded.

Our results exclude exact polynomial-time computation of Q (unconditionally) and of m_2 (assuming the structural conditions (A1)–(A3) and (R1) on a modified Robson reduction). They do *not* exclude:

- heuristic engines, including AlphaGo- or KataGo-style neural networks, which produce probabilistic estimates that can be wrong on adversarial positions such as S^*;
- exact algorithms in EXPTIME or PSPACE, which Robson's reduction shows are essentially the best we can ask for;
- polynomial-time algorithms for restricted classes of positions, such as decomposable endgames where the Berlekamp–Wolfe theory [3] of temperatures applies;
- exact algorithms under super-ko rules, where the complexity is an open question (only PSPACE-hardness is known [7], with no matching upper bound).

The first item is worth emphasizing. Practical Go engines achieve remarkable performance, but their outputs are heuristic estimates of m_1 and m_2, not certified values. Our theorem implies that exactness cannot be tightened down to a polynomial-time guarantee without separating P from EXPTIME, which is already a theorem.

7.2. **The standard 19×19 board.** For a fixed board size such as 19×19, the position set $\mathcal{P}_{19}$ is finite (Tromp and Farnëbäck [13] estimate $|\mathcal{P}_{19}| \approx 2.08 \times 10^{170}$), so a constant-size lookup table trivially computes m_2. The meaningful question is whether a *compact* formula—say a circuit of polynomial size in some surrogate parameter—exists. Existing techniques cannot decide this question: the relativization barrier [2], the natural proofs barrier [11], and the algebrization barrier [1] each rule out broad classes of methods. We therefore make no claim about the 19×19 case; all our results concern the asymptotic regime $n \to \infty$ of generalized Go.

7.3. **On the structural conditions (A1)–(A3) and (R1).** The conditions are stated as inputs to Theorem 6.1 rather than proved here, because doing so would require reproducing Robson's reduction in full detail. Instead, we record the construction sketches: (A1) is achieved by ranking ko-threat sizes geometrically; (A2) is achieved by padding the board with prefabricated two-eye live groups; (A3) is the standard tightness property of Robson's pipe and ladder gadgets, used implicitly in the original proof of Theorem 2.3; (R1) is the polynomial-size separation of accepting versus rejecting computations through the size of the main pipe group. Verifying all four properties for a fully written-out modified Robson reduction is a finite but technical exercise. We mark this verification as the natural followup to the present paper.

7.4. **Open problems.**

1. Verify (A1)–(A3) and (R1) for an explicit modified Robson reduction, removing the structural assumption from Theorem 6.1.
2. Extend Theorem 6.1 to super-ko rules. Robson's reduction depends on the basic ko rule for triple-ko encodings; a different reduction is needed.
3. Establish an analogue of Theorem 6.1 for the k-th best move for general k.
4. Settle whether Q admits a polynomial-size circuit family, i.e., whether $Q \in \mathrm{P/poly}$. By Karp–Lipton [9] and [4], a positive answer would imply a collapse of EXPTIME to the polynomial hierarchy, widely believed false but currently unproven.

References

[1] S. Aaronson and A. Wigderson. Algebrization: a new barrier in complexity theory. *ACM Transactions on Computation Theory*, **1**(1): Article 2, 2009.

[2] T. Baker, J. Gill, and R. Solovay. Relativizations of the P =?NP question. *SIAM Journal on Computing*, **4**(4): 431–442, 1975.

[3] E. R. Berlekamp and D. Wolfe. *Mathematical Go: Chilling Gets the Last Point*. A. K. Peters, Wellesley, MA, 1994.

[4] H. Buhrman, L. Fortnow, and T. Thierauf. Nonrelativizing separations. In *Proceedings of the 13th IEEE Conference on Computational Complexity*, pages 8–12, 1998.

[5] A. K. Chandra, D. C. Kozen, and L. J. Stockmeyer. Alternation. *Journal of the ACM*, **28**(1): 114–133, 1981.

[6] J. H. Conway. *On Numbers and Games*. Academic Press, London, 1976.

[7] M. Crâşmaru and J. Tromp. Ladders are PSPACE-complete. In *Computers and Games (CG 2000)*, volume 2063 of *Lecture Notes in Computer Science*, pages 241–249. Springer, 2000.

[8] J. Hartmanis and R. E. Stearns. On the computational complexity of algorithms. *Transactions of the American Mathematical Society*, **117**: 285–306, 1965.

[9] R. M. Karp and R. J. Lipton. Some connections between nonuniform and uniform complexity classes. In *Proceedings of the 12th Annual ACM Symposium on Theory of Computing*, pages 302–309, 1980.

[10] D. Lichtenstein and M. Sipser. GO is polynomial-space hard. *Journal of the ACM*, **27**(2): 393–401, 1980.

[11] A. A. Razborov and S. Rudich. Natural proofs. *Journal of Computer and System Sciences*, **55**(1): 24–35, 1997.

[12] J. M. Robson. The complexity of Go. In *Information Processing 83 (Proceedings of the IFIP 9th World Computer Congress)*, pages 413–417. North-Holland, Amsterdam, 1983.

[13] J. Tromp and G. Farnëbäck. Combinatorics of Go. In *Computers and Games (CG 2006)*, volume 4630 of *Lecture Notes in Computer Science*, pages 84–99. Springer, 2007. Updated counts at `https://tromp.github.io/go/legal.html`, accessed 2024.

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